Indices

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$
- $a^0 = 1$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $a^{-n} = \frac{1}{a^n}$
- $\frac{1}{a^{-n}} = a^n$
- $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$

Factorization

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a^2 - b^2) = (a-b)(a+b)$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $(a^3 + b^3) = (a+b)\left(a^2 - ab + b^2\right)$
- $(a^3 - b^3) = (a-b)\left(a^2 + ab + b^2\right)$
- $(x+a)(x+b) = x^2 + (a+b)x + ab$
- $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$
- $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

Quadratic Equation

$ax^2 + bx + c = 0$ is a quadratic equation a, b, c are real number and $a \neq 0$

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

  1. $b^2 - 4ac = 0\,;\quad x = -\frac{b}{2a},\ x = -\frac{b}{2a}$ both the roots are equal
  2. $b^2 - 4ac > 0\,;\quad x = \frac{-b + \sqrt{b^2 - 4ac}}{2a},\ x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ two distinct roots
  3. $b^2 - 4ac < 0\,;$ no real roots

Arithmetic Operations

- $ab + ac = a(b + c)$
- $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$
- $\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$
- $\frac{(a-b)}{(c-d)} = \frac{(b-a)}{(d-c)}$
- $\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{a \times d}{b \times c}$
- $a\left(\frac{b}{c}\right) = \frac{ab}{c}$
- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
- $\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$
- $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

Properties of Roots

- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $\sqrt[n]{a^n} = a$
- $\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$
- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

Ratio Proportion

Properties of ratio, $a : b$ and $c : d$ are the two given ratio

1. $ma : mb = a : b = \frac{a}{m} : \frac{b}{m}$
2. If $a \times d = b \times c$, then $a : b = c : d$
3. If $a \times d > b \times c$, then $a : b > c : d$
4. If $a \times d < b \times c$, then $a : b < c : d$

Properties of equal ratio

1. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$ ⇒ Invertendo
2. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$ ⇒ Alternendo
3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$ ⇒ Componendo
4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$ ⇒ Componendo
5. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$ ⇒ Dividendo
6. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c-d}{c-d}$ ⇒ Componendo Dividendo

Proportion If $\frac{a}{b} = \frac{c}{d}$ then a, b, c, d are said to be in proportion

$$\text{If} \quad \frac{a}{b} = \frac{c}{d} \quad \text{then} \quad a \times d = b \times c$$

Equal Ratios If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then,

$$\frac{a}{b} = \frac{a+c+e}{b+d+f}$$
$$\frac{c}{d} = \frac{a+c+e}{b+d+f}$$
$$\frac{e}{f} = \frac{a+c+e}{b+d+f}$$

Continued Proportion If $\frac{a}{b} = \frac{b}{c}$ then a, b, c are said to be in continued proportion. And if a, b, c are in continued proportion then,

$$\frac{a}{b} = \frac{b}{c} \quad \text{or} \quad b^2 = \frac{a}{c}$$
$$b = \sqrt{ac}$$

b is called **Geometric Mean** or **Mean**