Linear Equations in Two Variables

The general form of a linear equation in two variables x and y is
$ax + by + c = 0$, *and here* $a \neq 0, b \neq 0$

Solution of Linear equations in two variables by the method of Determinants
Let us consider two equations
$ax + by = e \ldots (1)$
$cx + dy = f \ldots (2)$ then

$$x = \frac{D_x}{D} \text{ and } y = \frac{D_y}{D}$$

Here,

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

$$D_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix} = e \times d - b \times f$$

$$D_y = \begin{vmatrix} a & e \\ c & f \end{vmatrix} = a \times f - e \times c$$

HCF and LCM of Polynomials

An algebric expression of the type $a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1} + a_nx^n$ *is called a polynomial if*
(i) $a_0, a_1, a_2 \cdots a_{n-1}, a_n$ *are real numbers*
(ii) n *is an integer which* ≥ 0

Test for divisiblity
Test for $(x-1)$**:**$(x-1)$ is a factor of polynomial in x if sum (additon) of all the coefficent of the polynomial is **zero**

Test for $(x+1)$**:**$(x+1)$ is a factor of polynomial in x if sum (additon) of all the coefficent in **even** power of x **is equal** to sum (additon) of all the coefficent in **odd** power of x

Relationship between HCF & LCM if there are two polynomial say $p(x)$ and $q(x)$ then
LCM of $\{\ p(x)$ and $q(x)\} \times$ HCF of $\{\ p(x)$ and $q(x)\}$ $= p(x) \times q(x)$

Rational Algebraic Expressions

we know that a number of the form $\frac{m}{n}$ where $n \neq 0$ is called a **rational number**.

$\therefore$ *An expression of the form* $\dfrac{p(x)}{qx}$ *where* $p(x), q(x)$ *are polynomials and* $q(x) \neq 0$, *is called* ***rational expression*** *or* ***rational algebraic expression***

$\Rightarrow$ if $p(x), q(x)$ and $r(x)$ are the polynomial and $q(x) \neq 0, r(x) \neq 0$ then

1. $\dfrac{p(x) \times r(x)}{q(x) \times r(x)} = \dfrac{p(x)}{q(x)}$
2. $-\left(\dfrac{p(x)}{q(x)}\right) = -\dfrac{p(x)}{q(x)} = \dfrac{-p(x)}{q(x)} = \dfrac{p(x)}{-q(x)}$

Addition & subtraction of two rational expressions (Denominator are equal)
Let $\dfrac{p(x)}{q(x)}$ & $\dfrac{r(x)}{q(x)}$ be any two rational expression then:

1. $\dfrac{p(x)}{q(x)} + \dfrac{r(x)}{q(x)} = \dfrac{p(x) + r(x)}{q(x)}$
2. $\dfrac{p(x)}{q(x)}$ - $\dfrac{r(x)}{q(x)} = \dfrac{p(x) - r(x)}{q(x)}$

Addition & subtraction of two rational expressions (Denominator are **not equal**)
Let $\dfrac{p(x)}{q(x)}$ & $\dfrac{r(x)}{s(x)}$ be any two rational expression then:

1. $\dfrac{p(x)}{q(x)} + \dfrac{r(x)}{s(x)} = \dfrac{p(x) \times s(x) + r(x) \times q(x)}{q(x) \times s(x)}$
2. $\dfrac{p(x)}{q(x)} - \dfrac{r(x)}{s(x)} = \dfrac{p(x) \times s(x) - r(x) \times q(x)}{q(x) \times s(x)}$

Multipication of Rational Expressions
If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then we we know that their product is given by $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$
similarly, if $\dfrac{p(x)}{q(x)}$ & $\dfrac{r(x)}{s(x)}$ are the two rational expression there product is given by
$\Rightarrow \dfrac{p(x)}{q(x)} \times \dfrac{r(x)}{s(x)} = \dfrac{p(x) \times r(x)}{q(x) \times s(x)}$

Division of Rational Expression
Let $\dfrac{p(x)}{q(x)}$ & $\dfrac{r(x)}{s(x)}$ be the two rational expression such that $r(x) \neq$ is non-zero rational expression then we have
$\Rightarrow \dfrac{p(x)}{q(x)} \div \dfrac{r(x)}{s(x)} = \dfrac{p(x)}{q(x)} \times \dfrac{s(x)}{r(x)} = \dfrac{p(x) \times s(x)}{q(x) \times r(x)}$

Quadratic Equations

The general form of a quadratic equation is $ax^2 + bx + c = 0$, *where a,b,c are real number and*

$a \neq 0$
sol. by perfect square method
⇒Make sure the coefficent of variable with index 2 is 1 (eg. $5x^2 - 4x - 2 = 0$, here the coefficent is 5 for x^2 hence first we shall divide both side by 5)
⇒Find the 3^{rd} term with formula
Third term=$\left(\frac{1}{2} \times \text{coefficent of } x\right)^2$
⇒ Add third term on both the side

sol. by formula method
Let $ax^2 + bx + c = 0$ be a quadratic equation, where where a,b,c are real numbers and $a \neq 0$ then solution by formula method is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case1:⇒ $b^2 - 4ac = 0$ i.e $b^2 = 4ac$
$x = -\frac{b}{2a}$, $x = -\frac{b}{2a}$ → both roots are equal
Case2:⇒ $b^2 - 4ac > 0$ i.e $b^2 > 4ac$ then the equation has two distinct root
$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$
Case 3:⇒ $b^2 - 4ac < 0$ then $\sqrt{b^2 - 4ac}$ is not real number and hence quadractic equation cannot have any real roots

Imp. Result

1. $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$
2. $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

Arthmetic Progressions(AP)

The general form of AP is
$t,\ t + d,\ t + 2d,\ t + 3d,\ \ldots$, here **t** is the first term and **d** is common difference,
The AP with first term 100 and common difference 50 is
$100, 150, 200, 250, 300 \ldots$

n^th (or the general) term of an AP, n^{th} of an arthemetic progression $t,\ t + d,\ t + 2d,\ t + 3d,\ \ldots$ is given by

$t_n = t + (n - 1)d$, here

✓ t_n is the nth term

✓ t is the first term

✓ n is the number of terms in an AP

✓ d is the common difference between the successive terms of an AP

Sum of the first n terms of an AP
Let $a,\ a+d,\ a+2d,\ a+3d,\ \ldots$ be an AP with total n number of terms
a denote first term,
t_n denote last term of AP
d denotes the common diff.
Let us denote sum upto n terms by S_n, we have
⇒ $S_n = \frac{n}{2}(a + t_n)$
⇒ $t_n = a + (n - 1)d$
∴ ⇒ $S_n = \frac{n}{2}[2a + (n - 1)d]$

Probability

Probability of an event
probability of an even A,written as P(A), is defined as
✓ P(A)=$\frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$
✓ probability of an impossible event is 0
✓ probability of a sure event is 1. Probability of any event will lie between 0 and 1, In general for any event A,we have
⇒P(A)=1-P(A′)
⇒P(A)+P(A′)=1
⇒P(A′)=1-P(A)
Here P(A′)denotes probability of not happening of an event A

Statistics

Mean of Raw Data
Mean of the values $x_1, x_2, x_3 \ldots, x_n$ is denoted by $\bar{x}$ and is given by

$$\bar{x} = \frac{x_1, x_2, x_3 \ldots, x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Assumed Mean Method for Calculating the Mean
A=an arbitary constant(usually A is chosen some where in the middle part of the given value) A is also called **assumed mean**
d_i=The reduced value, $d_i = x_i - A$ and is called **deviation** of x_i from A

$$\bar{d} = \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} \qquad \& \quad \textbf{Mean} = \bar{x} = A + \bar{d}$$

Mean of Grouped Data

1. Direct method $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
2. Assumed mean method $\bar{x} = A + \bar{d}$
3. Step-deviation method $\bar{x} = A + h.\bar{d}$
   (h is the width of the class intervals)

Remember $\bar{d}$ is calculated as $\bar{d} = \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$

Median

Formula for computing Median from grouped data

Median$=L + \frac{\frac{N}{2} - c.f.}{f} \times h$, where

L =Lower boundary of a median class
N =Total frequency
$c.f$=cumulative $f$requency of the class preceding the median class
f=$f$requency of the median class
h=Width of the median class

Mode

Formula for computing Mode from grouped data

Mode $= L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$

L =Lower boundary of a modal class
f_m=Frequency of the modal class
f_1=Frequency of the class coming before the modal class
f_2=Frequency fo the class coming after the modal class
h=Width of the modal class

Instalments

$\Rightarrow x_n = P_n \left(1 + \frac{R}{100}\right)^n$

P_n= Amount of n_{th} instalment.
P=Amount borrowed,cash price.
R=Rate of interest
A=Principle + interest=Amount due
x=Amount of installment

Similarity

Two $\triangle$ $'s$ are said to be similar if

1. *Their corresponding angles are equal*
2. *Their corresponding sides are in the same ratio (i.e sides are propotional)*

Ratio of area of $\triangle's$

Let A_1 and A_2 be the area, b_1, b_2 be the bases and h_1, h_2 be the heights of any two $\triangle's$ Then the ratio of there area is given as

1. $\frac{A_1}{A_2} = \frac{b_1 \times h_1}{b_2 \times h_2}$
2. $\frac{A_1}{A_2} = \frac{b_1}{b_2}$, if heights of two $\triangle's$ are equal
3. $\frac{A_1}{A_2} = \frac{h_1}{h_2}$, if bases of two $\triangle's$ are equal

Trigonometry

Trigonometric ratios of an acute angle

Sin θ	$\frac{\text{Opposite side of angle } \theta}{\text{Hypothenuse}}$
Cos θ	$\frac{\text{Adjacent side of angle } \theta}{\text{Hypothenuse}}$
Tan θ	$\frac{\text{Opposite side of angle } \theta}{\text{Adjacent side of angle } \theta}$
Cosec θ	$\frac{\text{Hypotenus}}{\text{Opposite side of angle } \theta}$
Sec θ	$\frac{\text{Hypotenus}}{\text{Adjacent side of angle } \theta}$
Cot θ	$\frac{\text{Adjacent side of angle } \theta}{\text{Opposite side of angle } \theta}$

Relationship between the trignometric ratios

sinAcosecA=1	sinA=$\frac{1}{\text{cosecA}}$	cosecA=$\frac{1}{\text{sinA}}$
cosAsecA=1	secA=$\frac{1}{\text{cosA}}$	cosA=$\frac{1}{\text{secA}}$
tanAcotA=1	tanA=$\frac{1}{\text{cotA}}$	cotA=$\frac{1}{\text{tanA}}$
tanA=$\frac{\text{sinA}}{\text{cosA}}$	cotA=$\frac{\text{cosA}}{\text{sinA}}$	

Ratios of angle

M. of $\angle^s \rightarrow$ $\downarrow$ Ratios of $\angle^s$	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
$\text{cosec}\,\theta$	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
$\cot\theta$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Identities

1. $sin^2\theta + cos^2\theta = 1$
2. $1 + tan^2\theta = sec^2\theta$
3. $1 + cot^2\theta = cosec^2\theta$

Trigonometric Ratios of Complementary Angles

1. $sin(90^\circ - \theta) = cos\ \theta$
2. $cos(90^\circ - \theta) = sin\ \theta$
3. $tan(90^\circ - \theta) = cot\ \theta$
4. $cot(90^\circ - \theta) = tan\ \theta$
5. $cosec(90^\circ - \theta) = sec\ \theta$
6. $sec(90^\circ - \theta) = cosec\ \theta$

Surface Areas & Volumes

1. **Cuboid** l:length, b:breadth, h:heigth
   (a) Curved Surface Area=$2h(l + b)$
   (b) Total Surface Area=$2(lb + bh + hl)$
   (c) Volume=lbh
2. **Cube** a: side of the cube
   (a) Curved Surface Area=$4a^2$
   (b) Total Surface Area=$6a^2$
   (c) Volume=a^3
3. **Right circular cylinder**
   r:radius of the base, h:height
   (a) Curved Surface Area=$2\pi rh$
   (b) Total Surface Area=$2\pi r(h + r)$
   (c) Volume=$\pi r^2 h$
4. **Cone** r:radius of the base
   h:height, l:slant height
   (a) Curved Surface Area=πrl
   (b) Total Surface Area=$2\pi r(l + r)$
   (c) Volume=$\frac{1}{3}\pi r^2 h$
   (d) By pythagoras theorem:$l^2 = h^2 + r^2$
5. **Sphere** r:radius
   (a) Surface Area=$4\pi r^2$
   (b) Volume=$\frac{4}{3}\pi r^3$
6. **Hemisphere** r:radius
   (a) Surface Area=$2\pi r^2$
   (b) Volume=$\frac{2}{3}\pi r^3$

Co-ordinate Geometry

Distance formula
Let $P(x_1, y_1), Q(x_2, y_2)$ be any two points
Distance formula=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
The distance of $P(x, y)$ from origin $O(0, 0)$ is given by $OP = \sqrt{(x^2 + y^2)}$
Section formula for internal division
coordinates of the point $P(x, y)$ which divides a line-segment joining the point $A(x_1, y_1)$, $B(x_2, y_2)$ internally in a given ratio $m : n$ are given by
$$P \equiv \left(\frac{mx_2 + nx_1}{m + n}\ ,\ \frac{my_2 + ny_1}{m + n}\right)$$
Section formula for external division
Coordinates of the point $P(x, y)$ dividing the seg.joining points $A(x_1, y_1), B(x_2, y_2)$ **externally** are given by $P \equiv \left(\frac{mx_2 - nx_1}{m - n}\ ,\ \frac{my_2 - ny_1}{m - n}\right)$
Mid-point formula
Point $P(x, y)$ is the midpoint of segment joining points $A(x_1, y_1)$, $B(x_2, y_2)$ its coordinates are given by $P \equiv \left(\frac{x_1 + x_2}{2}\ ,\ \frac{y_1 + y_2}{2}\right)$
Coordinates of Centroid
If $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are the vertices of $\triangle ABC$, $G(x, y)$ is the centroid then
$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}\ ,\ \frac{y_1 + y_2 + y_3}{3}\right)$$