Linear Equations in Two Variables

The general form of a linear equation in two variables x and y is $ax + by + c = 0$, here $a \neq 0, b \neq 0$

Solution of such equations by Determinants method

$$ax + by = e$$

$$cx + dy = f$$

(i) $\boldsymbol{x} = \dfrac{D_x}{D}$, (ii) $\boldsymbol{y} = \dfrac{D_y}{D}$ where

1. $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$
2. $D_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix} = e \times d - b \times f$
3. $D_y = \begin{vmatrix} a & e \\ c & f \end{vmatrix} = a \times f - e \times c$

HCF and LCM of Polynomials

An algebraic expression of the type $a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1} + a_nx^n$ *is called a polynomial if*

(i) $a_0, a_1, a_2 \cdots a_{n-1}, a_n$ are real numbers.

(ii) n is an integer which ≥ 0.

Test for divisibility

1. **Test for** $(x - 1)$**:**
   $(x - 1)$ is a factor of polynomial in x if sum (addition) of all the coefficient of the polynomial is **zero**
2. **Test for** $(x + 1)$**:**
   $(x + 1)$ is a factor of polynomial in x if sum (addition) of all the coefficient in **even** power of x **is equal** to sum (addition) of all the coefficient in **odd** power of x

Relationship between HCF & LCM
If there are two polynomial say $p(x)$ and $q(x)$ then

LCM of $\{\, p(x)$ and $q(x)\}$ $\times$ HCF of $\{\, p(x)$ and $q(x)\}$ = $p(x) \times q(x)$

Rational Algebraic Expressions

We know that a number of the form $\frac{m}{n}$ where $n \neq 0$ is called a **rational number**.
An expression of the form $\frac{p(x)}{qx}$ *where* $p(x), q(x)$ *are polynomials and* $q(x) \neq 0$, *is called* ***rational expression*** *or* ***rational algebraic expression***

Suppose $p(x), q(x)$ and $r(x)$ are the polynomials and $q(x) \neq 0, r(x) \neq 0$ then

(i) $\dfrac{p(x) \times r(x)}{q(x) \times r(x)} = \dfrac{p(x)}{q(x)}$

(ii) $-\left(\dfrac{p(x)}{q(x)}\right) = -\dfrac{p(x)}{q(x)} = \dfrac{-p(x)}{q(x)} = \dfrac{p(x)}{-q(x)}$

Addition & subtraction of two rational expressions
(When denominators are equal)

Let $\dfrac{p(x)}{q(x)}$ & $\dfrac{r(x)}{q(x)}$ be any two rational expressions then:

1. $\dfrac{p(x)}{q(x)} + \dfrac{r(x)}{q(x)} = \dfrac{p(x) + r(x)}{q(x)}$
2. $\dfrac{p(x)}{q(x)} - \dfrac{r(x)}{q(x)} = \dfrac{p(x) - r(x)}{q(x)}$

Addition & subtraction of two rational expressions
(When denominators are unequal)

Let $\dfrac{p(x)}{q(x)}$ & $\dfrac{r(x)}{s(x)}$ be any two rational expressions then:

1. $\dfrac{p(x)}{q(x)} + \dfrac{r(x)}{s(x)} = \dfrac{p(x) \times s(x) + r(x) \times q(x)}{q(x) \times s(x)}$
2. $\dfrac{p(x)}{q(x)} - \dfrac{r(x)}{s(x)} = \dfrac{p(x) \times s(x) - r(x) \times q(x)}{q(x) \times s(x)}$

Multiplication of Rational Expressions

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then we we know that their product is given by $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ similarly, if $\frac{p(x)}{q(x)}$ and $\frac{r(x)}{s(x)}$ are the two rational expression there product is given by

$$\frac{p(x)}{q(x)} \times \frac{r(x)}{s(x)} = \frac{p(x) \times r(x)}{q(x) \times s(x)}$$

Division of Rational Expression

Let $\dfrac{p(x)}{q(x)}$, $\dfrac{r(x)}{s(x)}$ be the two rational expressions such that $r(x) \neq$ is non-zero rational expression then we have

$$\frac{p(x)}{q(x)} \div \frac{r(x)}{s(x)} = \frac{p(x)}{q(x)} \times \frac{s(x)}{r(x)} = \frac{p(x) \times s(x)}{q(x) \times r(x)}$$

Quadratic Equations

The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a , b , c are real number and $a \neq 0$

Solution by perfect square method

1. Make sure the coefficient of variable with index 2 is 1 (eg. $5x^2 - 4x - 2 = 0$, here the coefficient is 5 for x^2 hence first we shall divide both side by 5)
2. Find the 3^{rd} term with formula **Third term**=$\left(\frac{1}{2} \times \text{coefficient of } x\right)^2$
3. Add third term on both the side

Solution by formula method
Let $ax^2 + bx + c = 0$ be a quadratic equation, where where a , b , c are real numbers and $a \neq 0$ then solution by formula method is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case1: $b^2 - 4ac = 0$ i.e $b^2 = 4ac$

$$x = -\frac{b}{2a} \quad , \quad x = -\frac{b}{2a}, \text{both the roots are equal}$$

Case2: $b^2 - 4ac > 0$ i.e $b^2 > 4ac$ then the equation has two distinct roots

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad , \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Case3: $b^2 - 4ac < 0$ then $\sqrt{b^2 - 4ac}$ is not a real number and hence quadratic equation cannot have any real roots

Imp. Result

1. $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$
2. $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

Arithmetic Progressions(AP)

General form of the AP

$$t,\ t + d,\ t + 2d,\ t + 3d, \ldots$$

(i) t is the ***first term*** (ii) d is ***common difference***
n^{th} term of an AP $t,\ t + d,\ t + 2d,\ t + 3d, \ldots$ is given by

$$t_n = t + (n - 1)d$$

1) t_n is n^{th} term
2) t is the first term
3) n is the number of terms in an AP
4) d is the common difference between the successive terms of an AP

Sum of the first n terms of an AP
Let $a,\ a + d,\ a + 2d,\ a + 3d,\ \ldots$ be an AP with n number of terms
a denote first term.
t_n denote last term of AP.
d denotes the common difference.
Let us denote sum up to n terms by S_n, we have

1. $S_n = \frac{n}{2}(a + t_n)$
2. $t_n = a + (n - 1)d$

$$S_n = \frac{n}{2}\Big[2a + (n - 1)d\Big]$$

Probability

Probability of an event

probability of an event A, written as $P(A)$, is defined as
$P(A) = \frac{\text{Number of outcomes favorable to } A}{\text{Total number of possible outcomes}}$
probability of an impossible event is 0
probability of a sure event is 1.
Probability of any event will lie between 0 and 1.

In general for any event A, we have

1. $P(A) = 1 - P(A')$
2. $P(A) + P(A') = 1$
3. $P(A) = 1 - P(A)$

Note: $P(A')$ denotes probability of ***not*** happening of an event A

Statistics

Mean of Raw Data

Mean of the values $x_1, x_2, x_3 \ldots, x_n$ is denoted by $\bar{x}$ and is given by

$$\bar{x} = \frac{x_1 + x_2 + x_3 \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Assumed Mean Method for Calculating the Mean

A is an arbitrary constant(usually A is chosen some where in the middle part of the given value) A is also called **assumed mean**. d_i=The reduced value, $d_i = x_i - A$ and is called **deviation** of x_i from A

$$\bar{d} = \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} \qquad \& \quad \textbf{Mean} = \bar{x} = A + \bar{d}$$

Mean of Grouped Data

1. Direct method $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
2. Assumed mean method $\bar{x} = A + \bar{d}$
3. Step-deviation method $\bar{x} = A + h.\bar{d}$
   (h is the width of the class intervals)

Median

Formula for computing Median from grouped data

$$\textbf{Median} = L + \frac{\frac{N}{2} - c.f.}{f} \times h$$

1) L is lower boundary of a median class
2) N is total frequency

3) $c.f$ is cumulative frequency of the class preceding the median class
4) f is frequency of the median class
5) h is width of the median class

Mode

Formula for computing Mode from grouped data

$$\textbf{Mode} = L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

1) L is lower boundary of a modal class
2) f_m is frequency of the modal class
3) f_1 is frequency of the class coming before the modal class
4) f_2 is frequency of the class coming after the modal class
5) h is width of the modal class

Similarity

Ratio of the areas of two triangles

Let A_1 and A_2 be the areas, b_1, b_2 be the bases and h_1, h_2 be the heights of any two $\triangle's$ Then the ratio of there area is given as

1. $\dfrac{A_1}{A_2} = \dfrac{b_1 \times h_1}{b_2 \times h_2}$
2. $\dfrac{A_1}{A_2} = \dfrac{b_1}{b_2}$, if heights of two $\triangle's$ are equal
3. $\dfrac{A_1}{A_2} = \dfrac{h_1}{h_2}$, if bases of two $\triangle's$ are equal

Trigonometry

Trigonometric ratios of an acute angle

$\sin\theta$	$\dfrac{\text{opposite side of the angle } \theta}{\text{hypotenuse}}$	$\operatorname{cosec}\theta$	$\dfrac{\text{hypotenuse}}{\text{opposite side of the angle } \theta}$
$\cos\theta$	$\dfrac{\text{adjacent side of the angle } \theta}{\text{hypotenuse}}$	$\sec\theta$	$\dfrac{\text{hypotenuse}}{\text{adjacent side of the angle } \theta}$
$\tan\theta$	$\dfrac{\text{opposite side of the angle } \theta}{\text{adjacent side of the angle } \theta}$	$\cot\theta$	$\dfrac{\text{adjacent side of the angle } \theta}{\text{opposite side of the angle } \theta}$

M. of $\angle^s$ $\rightarrow$ $\downarrow$ Ratios of $\angle^s$	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
$\operatorname{cosec}\theta$	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
$\cot\theta$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Trigonometric Ratios of Complementary Angles

1. $\sin(90^\circ - \theta) = \cos\theta$
2. $\cos(90^\circ - \theta) = \sin\theta$
3. $\tan(90^\circ - \theta) = \cot\theta$
4. $\cot(90^\circ - \theta) = \tan\theta$
5. $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$
6. $\sec(90^\circ - \theta) = \operatorname{cosec}\theta$

Surface Areas & Volumes

1. **Cuboid** l is the length, b is the breadth, h is the height
   (a) Curved Surface Area $= 2h(l + b)$
   (b) Total Surface Area $= 2(lb + bh + hl)$
   (c) Volume $= lbh$
2. **Cube** a is measure the side of the cube
   (a) Curved Surface Area $= 4a^2$
   (b) Total Surface Area $= 6a^2$
   (c) Volume $= a^3$
3. **Right circular cylinder** r is radius of the base, h is the height
   (a) Curved Surface Area $= 2\pi rh$
   (b) Total Surface Area $= 2\pi r(h + r)$
   (c) Volume $= \pi r^2 h$
4. **Cone** r is radius of the base, h is the height, l is the slant height
   (a) Curved Surface Area $= \pi rl$
   (b) Total Surface Area $= 2\pi r(l + r)$
   (c) Volume $= \frac{1}{3}\pi r^2 h$
   (d) By Pythagoras theorem $l^2 = h^2 + r^2$
5. **Sphere** r is the radius
   (a) Surface Area $= 4\pi r^2$
   (b) Volume $= \frac{4}{3}\pi r^3$
6. **Hemisphere** r is the radius
   (a) Surface Area $= 2\pi r^2$
   (b) Volume $= \frac{2}{3}\pi r^3$

Coordinate Geometry

1. Distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. Distance between a point $P(x, y)$ and origin $O(0, 0)$

$$PO = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

3. Coordinates of point P, dividing the line-segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ ***internally*** in the ratio $m : n$ are given by *section formula*

$$P \equiv \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right) \tag{1}$$

Special Case

(a) The mid-point of the line-segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ divides the line-segment in the ratio $1 : 1$. Hence, putting $m = 1$ and $n = 1$ in equation (**??**)

$$P \equiv \left(\frac{x_1 + x_2}{m + n}, \frac{y_1 + y_2}{m + n}\right) \tag{2}$$

4. Coordinates of point P, dividing the line-segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ ***externally*** in the ratio $m : n$ are given by

$$P \equiv \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

5. $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the coordinates of the vertices of a $\triangle ABC$ and $G(x, y)$ is the centroid of the triangle

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$