

## Indices

$$(1) a^m \times a^n = a^{m+n}$$

$$\text{eg. } 10^3 \times 10^2 = 10^{3+2} = 10^5$$

$$(2) a^m \div a^n = a^{m-n} ; m > n , a \neq 0$$

$$\text{eg. } 8^4 \div 8^2 = 8^{4-2} = 8^2$$

$$(3) (a^m)^n = a^{m \times n}$$

$$\text{eg. } (6^4)^2 = 6^{4 \times 2}$$

$$(4) (a \times b)^m = a^m \times b^m$$

$$\text{eg. } (5 \times 3)^6 = 5^6 \times 3^6$$

$$(5) \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} ; b \neq 0$$

$$\text{eg. } \left(\frac{5}{4}\right)^8 = \frac{5^8}{4^8}$$

$$(6) a^0 = 1 \quad \text{eg. } 7^0 = 1$$

$$(7) a^{-m} = \frac{1}{a^m} \quad \text{eg. } 5^{-3} = \frac{1}{5^3}$$

## Area of Triangle

To find the area of triangle, given the base and height

$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$

To find the area of triangle when the length of all its three sides are given

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

In the above formula  $a, b, c$  are the sides of a triangle and  $s$  is the semi-perimeter of triangle.  $s = \frac{1}{2}(a+b+c)$

## Identities-Expansion, Factors

$$(1) (a+b)^2 = a^2 + 2ab + b^2$$

$$(2) (a-b)^2 = a^2 - 2ab + b^2$$

$$(3) (a+b)(a-b) = a^2 - b^2$$

$$(4) (x+a)(x+b) = (x+a) \times x + (x+a) \times b \\ = x^2 + ax + bx + ab \\ = x^2 + (a+b)x + ab$$

$$(5) (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(6) (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(7) (a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$

$$(8) (a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

## Arc of a Circle

### Circumference of the circle

$$(1) C = 2\pi r \quad \text{i.e. } 2 \times \pi \times r$$

Formula (1) is used to find the circumference when radius is known

$$(2) C = \pi d \quad \text{i.e. } \pi \times d$$

Formula (2) is used to find the circumference when diameter is known

In the above formulas value of  $\pi$  can be taken as  $\frac{22}{7}$ , if it's not given

### Length of an Arc

$$\text{Length of an arc} = \frac{\theta}{360} \times 2\pi r$$

$\theta$  is the angular measure and  $r$  is the radius of the circle

## Area of a Circle

$$\text{Area of a Circle} = \pi \times r^2 \quad \text{i.e. } \pi \times r \times r$$

Here  $r$  is the radius of the circle

### Area of the sector of circle

$$\text{Area of the sector of circle} = \frac{\theta}{360} \times \pi r^2$$

Here  $\theta$  is the angular measure of the arc and  $r$  is the radius

## Compound Interest

$$I = \frac{P \times N \times R}{100}$$

$I$  = Simple Interest

$P$  = Principal

$N$  = Number of years

$R$  = Rate of Interest

$A = P + I$  ;  $A$  is the Amount and it includes Principal and Interest accrued

Amount( $A$ ) by compound interest

$$= \text{Principal} \left(1 + \frac{\text{Rate}}{100}\right)^{\text{Period}}$$

$$\text{i.e. } (A) = P \left(1 + \frac{R}{100}\right)^N$$

## Volume and Surface Area

### Cylinder

$$\text{Volume of a Cylinder} = \pi r^2 h \quad \text{i.e. } \pi \times r \times r \times h$$

Curved Surface area of a cylinder

$$= 2\pi r h \quad \text{i.e. } 2 \times \pi \times r \times h$$

Total surface area of a cylinder

$$= 2\pi r(h+r) \quad \text{i.e. } 2 \times \pi \times r \times (h+r)$$

In the formula  $r$  is the radius of the base of a cylinder and  $h$  is the height

**Cone**

Volume of the cone =  $\frac{1}{3} \times \pi r^2 h$

Curved Surface Area of the cone =  $\pi r l$

Total Surface Area of the cone =  $\pi r (l + r)$

In the formula  $r$  is radius of the base of the cone and  $h$  is the height and  $l$  is the slant height

if any two  $l, r, h$  are given and you need to find the third one, then you use the formula

$l^2 = h^2 + r^2$

**Sphere**

Volume of the sphere =  $\frac{4}{3} \times \pi r^3$

Surface area of the sphere =  $4\pi r^2$

$r$  is the radius of the sphere

**Sets**

**Complement of set**

- 1) For a set A if  $A'$  is its complement then  $(A')' = A$
  - 2) If U is an universal set and  $U'$  is its complement then  $U' = \phi$
  - 3) if  $\phi$  denotes an empty set then  $\phi' = U$
- where  $\phi'$  is complement of  $\phi$

**Properties of union of sets**

Here A and B are two subsets of universal set U then

- 1)  $A \cup B = B \cup A$
- 2) If  $A \subseteq B$  then  $A \cup B = B$
- 3) If  $A \subseteq A \cup B$
- 4)  $A \cup A' = U$
- 5)  $A \cup A = A$
- 6)  $A \cup \phi = A$

**Properties of intersection of sets**

- 1)  $A \cap B = B \cap A$
- 2) If  $A \subseteq B$  then  $A \cap B = A$
- 3)  $A \cap B \subseteq A$  and  $A \cap B \subseteq B$
- 4)  $A \cap A' = \phi$
- 5)  $A \cap A = A$
- 6)  $A \cap \phi = \phi$

**Number of elements in a set**

Number of element in the set A is denoted by  $n(A)$  we have

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

if set A and B are disjoint then  $n(A \cap B) = 0$  and we have

$n(A \cup B) = n(A) + n(B)$

**Real Numbers**

If a number is of the form  $\frac{p}{q}$  where  $p$  and  $q$  are integers and denominator  $q \neq 0$ , then the number is called a rational number

Set of all rational numbers is denoted by the letter Q and

$\therefore Q = \left\{ \frac{p}{q} \mid p, q \in I \text{ and } q \neq 0 \right\}$

**Equality relation:**  $\frac{p}{q}$  and  $\frac{r}{s}$  are any two rational numbers and

if  $\frac{p}{q} = \frac{r}{s}$ , then  $ps = qr$  and conversely if  $ps = qr$  then  $\frac{p}{q} = \frac{r}{s}$

**Order relation:**  $\frac{p}{q}$  and  $\frac{r}{s}$  are any two rational numbers with both denominators  $q > 0$  and  $s > 0$  then

if  $\frac{p}{q} > \frac{r}{s}$ , then  $ps > qr$  and conversely if  $ps > qr$  then  $\frac{p}{q} > \frac{r}{s}$

**Properties of rational number**

if  $a, b, c$  are any rational numbers then

$a + b = b + a$	Commutative property
$(a + b) + c = a + (b + c)$	Associative property
$a + 0 = 0 + a = a$	Additive identity
$a + (-a) = (-a) + a = 0$	Additive inverse
$a \times b = b \times a$	Commutative property
$(a \times b) \times c = a \times (b \times c)$	Associative property
$a \times 1 = 1 \times a = a$	Multiplicative identity
$a \times \frac{1}{a} = \frac{1}{a} \times a = 1$	Multiplicative inverse
$a \times (b + c) = a \times b + a \times c$	Distributive property

**Properties of real number**

1) If  $a$  and  $b$  are any two real numbers, then only one of the following relation is true

- (i)  $a = b$       (ii)  $a < b$       (iii)  $a > b$

2) If  $a < b$  and  $b < c$  then  $a < c$

3) If  $a < b$  then  $a + c < b + c$

4) Let  $a < b$  then

(i) If  $c > 0$  then  $ac < bc$  (ii) If  $c < 0$  then  $ac > bc$

If  $x$  is a real number absolute value of  $x$  is denoted by  $|x|$  and is defined as

$|x| = x$  for  $x > 0$

$= 0$  for  $x = 0$

$= -x$  for  $x < 0$  and if  $|x| = a$  then  $x = \pm a$

**Surds**

A number of the form  $\sqrt[n]{a}$  is said to be a surd if

$\sqrt[n]{a}$  is a natural number and  $n \neq 1$

$\sqrt[n]{a}$  is a positive rational number

$\sqrt[n]{a}$  is an irrational number

**Laws of Surd**

1)  $(\sqrt[n]{a})^n = a$       e.g.  $(\sqrt[4]{10})^4 = 10$

2)  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$       e.g.  $\sqrt[3]{20} \cdot \sqrt[3]{4} = \sqrt[3]{80}$

3)  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$       e.g.  $\frac{\sqrt[7]{10}}{\sqrt[7]{19}} = \sqrt[7]{\frac{10}{19}}$

4)  $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$  e.g.  $\sqrt[3]{\sqrt[5]{72}} = \sqrt[15]{72} = \sqrt[5]{\sqrt[3]{72}}$

**Forms of Surd**

1) Pure surds : A surd of the form  $\sqrt[n]{a}$  is called a pure surd  
e.g.  $\sqrt[3]{19}$  is pure surd

2) Mixed surds : A surd of the form  $m \times \sqrt[n]{a}$  where  $m (\pm 1)$  is a rational number, is called a mixed surd  
e.g.  $8\sqrt[3]{27}$  is mixed surd

3) Similar Surd  
 $\Rightarrow$  The surds of the form  $p\sqrt[n]{a}$  and  $q\sqrt[n]{a}$ , where  $p$  and  $q$  are rational numbers are called similar surds  
e.g.  $\sqrt{4}, \sqrt[3]{4}, \frac{7}{8}\sqrt{4}$  are similar surds

### Simplest form of a surd

A Surd  $\sqrt[n]{a}$  is said to be in its simplest form if  
 ✓The radicand  $a$  has no factor which is  $n^{th}$  power of a rational number  
 ✓The radicand  $a$  is not fraction  
 ✓ $n$  is the least order

### Comparison of surds

Suppose  $\sqrt[n]{a}$  and  $\sqrt[n]{b}$  are two surds of same order( $n$ ) then they can be compared by comparing there radicand ( $a, b$  in the e.g)

- i)  $\sqrt[n]{a} = \sqrt[n]{b}$  if  $a = b$
- ii)  $\sqrt[n]{a} > \sqrt[n]{b}$  if  $a > b$
- iii)  $\sqrt[n]{a} < \sqrt[n]{b}$  if  $a < b$

**Rationalization of surds:** If we multiply two surds and the product we get is a rational number, then we say each surd is a rationalizing factor of the other surd

## Factorization of algebraic expression

Factorization of an algebraic expression of the form

$$a^3 + b^3 + c^3 - 3abc$$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

- (i) If  $a + b + c = 0$  then  $a^3 + b^3 + c^3 = 3abc$
- (ii) If  $a + b + c \neq 0$  and  $a^3 + b^3 + c^3 - 3abc = 0$  then  $a = b = c$

## Polynomials

An algebraic expression is called a polynomial if

✓It's of the form  $a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n$

✓ $a_0, a_1, a_2 \dots a_{n-1}, a_n$  are real numbers

✓ $n$  is a non-negative integer

a polynomial in  $x$  is denoted by  $p(x)$ ,

If we arrange terms of polynomial such that power of  $x$  are in ascending or are in descending order, then we say polynomial is in the **standard form**

**Degree of a polynomial:** Suppose a polynomial is in  $x$ , then the highest power(index) of  $x$  is called the degree of the polynomial

### Types of Polynomial

Monomial	One term	e.g $3, 10x, 12x^3$
Binomial	Two terms	e.g $4x + 12x^3$
Trinomial	Three terms	e.g $3 + 10x + 4x^2$

**Zero polynomial:** If all coefficient of a polynomial are zero it's called a zero polynomial

e.g  $0 + 0x, 0x + 0x^3$

**Constant polynomial:** if a polynomial  $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} + a_nx^n$  is such that  $a_1 = a_2 = a_3 = \dots a_{n-1} = a_n = 0$  then  $P(x)$  is called constant polynomial i.e  $p(x) = a_0 + 0 + 0 + 0 \dots = a_0$   
 $p(x) = 12, p(x) = -2$  are examples constant polynomial

## Ratio and proportion

### Properties of ratio

- 1)  $ma : mb = a : b = \frac{a}{m} : \frac{b}{m}$
- 2) if  $a : b$  and  $c : d$  are the two given ratio then

- $\Rightarrow$  if  $a \times d = b \times c$  then  $a : b = c : d$
- $\Rightarrow$  if  $a \times d > b \times c$  then  $a : b > c : d$
- $\Rightarrow$  if  $a \times d < b \times c$  then  $a : b < c : d$

### Properties of equal ratios

- $\Rightarrow$  if  $\frac{a}{b} = \frac{c}{d}$  then  $\frac{b}{a} = \frac{d}{c}$   $\rightarrow$  Invertendo
- $\Rightarrow$  if  $\frac{a}{b} = \frac{c}{d}$  then  $\frac{a}{c} = \frac{b}{d}$   $\rightarrow$  Alternedo
- $\Rightarrow$  if  $\frac{a}{b} = \frac{c}{d}$  then  $\frac{a+b}{b} = \frac{c+d}{d}$   $\rightarrow$  Componendo
- $\Rightarrow$  if  $\frac{a}{b} = \frac{c}{d}$  then  $\frac{a-b}{b} = \frac{c-d}{d}$   $\rightarrow$  Dividendo
- $\Rightarrow$  if  $\frac{a}{b} = \frac{c}{d}$  then  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$   $\rightarrow$  Componendo-Divedendo

**Theorem on equal ratios:** if  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$  then

$$\sqrt{\frac{a}{b}} = \frac{a+c+e}{b+d+f}$$

$$\sqrt{\frac{c}{d}} = \frac{a+c+e}{b+d+f}$$

$$\sqrt{\frac{e}{f}} = \frac{a+c+e}{b+d+f}$$

i.e each ratio =  $\frac{a+c+e}{b+d+f}$

**Proportion:** if  $\frac{a}{b} = \frac{c}{d}$  then  $a, b, c, d$  are said to be in proportion

if  $\frac{a}{b} = \frac{c}{d}$  then  $a \times d = b \times c$

**Continued proportion:** if  $\frac{a}{b} = \frac{b}{c}$  then  $a, b, c$  are said to be in continued proportion

we have if  $a, b, c$  are in continued proportion then

$$\frac{a}{b} = \frac{b}{c} \quad \text{i.e } b^2 = \frac{a}{c}$$

$\therefore b = \sqrt{ac}$   $b$  is called geometric mean or mean proportional to  $a$  and  $c$

## Statistic

### Mean

Mean of raw data : If  $x_1, x_2, \dots x_n$  are given observations all are numbers then we defined their mean as  $\bar{x}$ , and  $\bar{x} = \frac{x_1+x_2+x_n}{n}$

### Mean of ungrouped data

$$\bar{x} = \frac{f_1x_1 + f_2x_2 \dots + f_nx_n}{f_1 + f_2 \dots + f_n} \quad \text{i.e } \bar{x} = \frac{1}{N} \sum f_i x_i \text{ where } N = \sum f_i$$

Here,  $x_1, x_2, \dots x_n$  are observations and  $f_1, f_2, f_3 \dots f_n$  are frequencies

### Median

Suppose  $N$  is the number of observation made, and they are arranged in either ascending or in descending order of magnitudes then

1. If  $N$  is Odd then Median =  $\left(\frac{N+1}{2}\right)^{th}$  term
2. if  $N$  is Even then Median = A.M of  $\left(\frac{N}{2}\right)^{th}$  and  $\left(\frac{N+1}{2}\right)^{th}$  terms

**Mode:**The observation(s) which has a maximum number of frequency is called a mode

## Percentage profit & loss

- 1) Profit = S.P - C.P
- 2) Loss = C.P - S.P

$$3) \text{ Profit percentage} = \frac{P_{profit}}{C.P} \times 100$$

$$4) \text{ Loss percentage} = \frac{Loss}{C.P} \times 100$$

## Discount Rebate & Commission

1. Discount = printed price  $\times$  rate of discount

2. Selling price = printed price - discount

3. Percentage of discount =  $\frac{\text{discount}}{\text{printed price}} \times 100$

## Trigonometry

### Trigonometric ratios of an acute angle

Sin $\theta$	$\frac{\text{Opposite side of angle } \theta}{\text{Hypotenuse}}$
Cos $\theta$	$\frac{\text{Adjacent side of angle } \theta}{\text{Hypotenuse}}$
Tan $\theta$	$\frac{\text{Opposite side of angle } \theta}{\text{Adjacent side of angle } \theta}$
Cosec $\theta$	$\frac{\text{Hypotenuse}}{\text{Opposite side of angle } \theta}$
Sec $\theta$	$\frac{\text{Hypotenuse}}{\text{Adjacent side of angle } \theta}$
Cot $\theta$	$\frac{\text{Adjacent side of angle } \theta}{\text{Opposite side of angle } \theta}$

### Relationship between the trigonometric ratios

$\sin A \operatorname{cosec} A = 1$	$\sin A = \frac{1}{\operatorname{cosec} A}$	$\operatorname{cosec} A = \frac{1}{\sin A}$
$\cos A \sec A = 1$	$\cos A = \frac{1}{\sec A}$	$\sec A = \frac{1}{\cos A}$
$\tan A \cot A = 1$	$\tan A = \frac{1}{\cot A}$	$\cot A = \frac{1}{\tan A}$
$\tan A = \frac{\sin A}{\cos A}$	$\cot A = \frac{\cos A}{\sin A}$	

### Ratios of angle

M. of $\angle^s \rightarrow$ $\downarrow$ Ratios of $\angle^s$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
sin $\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos $\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan $\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
cosec $\theta$	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec $\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
cot $\theta$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

### Perimeters(P) and Areas(A) of some plane fig's

1. Triangle:  $a, b, c$  are sides of  $\triangle$ ,  $h$  is the ht

(a) Perimeter =  $a + b + c$

(b) Area =  $\frac{1}{2} \times \text{base} \times \text{height}$

(c) By Heron's formula

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)}$$

$$\text{here } s = \frac{a+b+c}{2}$$

2. Equilateral  $\triangle$ : Each side= $a$

(a) Perimeter =  $a + a + a = 3a$

(b) Area =  $\frac{\sqrt{3}}{4} \times a^2$

3. Square: Each side= $a$

(a) Perimeter =  $a + a + a + a = 4a$

(b) Area =  $a \times a = a^2$

4. Rectangle:  $l$  is length &  $b$  is breadth

(a) Perimeter =  $2 \times (l + b)$

(b) Area=length  $\times$  breadth i.e  $l \times b$

5. Parallelogram: $a, b$  are sides of  $\parallel^m$  and  $h$  is ht.

(a) Perimeter =  $2 \times (a + b)$

(b) Area=base  $\times$  height i.e  $b \times h$

6. Trapezium:  $a, b, c, d$  are sides,  $h$  is height, here sides  $a$  &  $b$  are opp.  $\parallel$  sides

(a) Perimeter =  $a + b + c + d$

(b) Area= $\frac{1}{2} \times (\text{sum of the lengths of } \parallel \text{ sides}) \times (\text{height})$   
 $= \frac{1}{2} (a + b) \times h$

7. Rhombus:  $l$  is the measure of all sides and  $d_1, d_2$  are the diagonals.

(a) Perimeter =  $4 \times l$

(b) Area= $\frac{1}{2} \times (\text{products of the diagonals})$   
 $= \frac{1}{2} (d_1 \times d_2)$

8. circle: $r$  is the radius.

(a) Perimeter = Circumference =  $2\pi r$

(b) Area =  $\pi r^2$